

# **Investigation of the Huppert and Sparks Convective Magma Chamber Theory as a Valid Model for Geologic Processes**

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## **Abstract**

In 1988 Huppert and Sparks published two papers, “The generation of granitic magmas by intrusion of basalt into continental crust”, and “Melting the roof of a chamber containing a hot, turbulently convecting fluid”, both pertaining to experiments they conducted (Huppert, 1986). They constructed a theory in which a basaltic magma is intruded into continental crust at high sufficiently temperature, that the basaltic magma chamber vigorously convects. Sufficient heat is transferred to the roof rock causing felsic material above the magma chamber to melt. It is their assertion that this region obtains enough heat from below to cause convection of the newly formed silicic magma chamber. This process can occur in timescales from  $10^2$  to  $10^7$  yrs. Examination of this theory, however, shows that although this concept is worthy of consideration, the comparison of their experiments to the geologic process is inappropriate.

## **Introduction.**

The subject of how magma chambers form, evolve and solidify has been the source of much controversy in the geological/ geophysical community for many years. Speculations on the dominant thermal process vary from conductive (Brandeis and Marsh, 1989, Bergantz and Dawes 1994) to convective regimes (Huppert 1986, Cardoso and Woods 1996). Additionally, the question of whether a silicic chamber can form separately, or is the result of heat transfer from an underlying basaltic magma chamber has also been a topic of much debate. In the later 1980's Huppert and Sparks produced a series of articles concerning this very subject. The culmination of their conclusions was conveyed in the article “The generation of granitic magmas by intrusion of basalt into continental crust” (henceforth referred to as H&S1988a). The mathematical details of this article are in its concurrently written paper “Melting the roof of a chamber containing a hot, turbulently convecting fluid” (henceforth referred to as H&S1988b). These papers are both highly cited.

Their model is a convecting basaltic magma chamber emplaced within continental crust, which then melts the overlying crust producing a convecting felsic magma chamber. The experiments they performed employed a Polyethylene glycol wax (approximate molecular weight of 1000) above an aqueous solution of sodium nitrate. However, upon careful examination of the comparison of experiment to nature, this author has found several inconsistencies in their findings, which will be examined in this paper. It is suggested here that these discrepancies warrant careful reexamination of the application of this model.

## **Background**

### *Laboratory Experiments*

The experiments involved a hot aqueous solution of approximately 18%  $\text{NaNO}_3$  injected below a wax roof with a thickness of 15 cm, suspended at the top of a 20 x 20 x 40cm perspex container. The concentration of the  $\text{NaNO}_3$  solution was not given by Huppert and Sparks, but was approximated by the density of the solution, given in a photograph (Fig. 1) of H&S1988a, and a table presented in the CRC Handbook of Chemistry and Physics (65th Ed.) of  $\text{NaNO}_3$  solutions. Enough of the aqueous solution was introduced such that the perspex container was completely filled. The melting temperature of the PEG 1000 used is 37-40°C and the initial temperature of the hot aqueous solution was around 70°C. The density of the wax decreased with a linear trend from 1.11 g/cm<sup>3</sup> at 40°C to 1.09 g/cm<sup>3</sup> at 70°C. The density for the aqueous solution was 1.13 g/cm<sup>3</sup> (according to the figure). The authors indicated that the melt formed a separate layer between the roof and the hot fluid, that the underlying layer was initially

quiescent, transferring heat by conduction alone, and then subsequently convection occurred.

### *Theory*

In the H&S1988a paper, Huppert and Sparks presented a theoretical description of these layered convecting magma chambers based on the data they obtained from their experiments. The details of this report are in the H&S1988b paper. A brief outline of this model will be included here. The nomenclature for this is provided in appendix A.

Their theory considers a hot basaltic magma, sill size, emplaced into continental crust which initially has a uniform temperature. The basaltic magma starts convecting, which effectively focuses sufficient heat from below that the roof rock is melted. At first the silicic sheet is too thin to convect, though after a finite period convection does occur and a well stirred silicic chamber is formed above. As the basaltic magma layer cools and crystallizes it loses heat, and eventually convection ceases. The basalt continues to provide heat to the silicic chamber via conduction. It is the assertion of Huppert and Sparks that the silicic chamber has gained enough heat such that convection continues for an extended period of time. Ultimately, the silicic magma chamber gains enough crystallinity such that convection ceases there as well.

Utilizing the denotation from appendix A, we have a heat flux from the convecting basaltic magma to the silicic magma, given by:

$$F_b = \rho_b c_b J_b (T_b - T_i)^{4/3} \quad (1)$$

where the four-thirds exponent is a result of a scaling relationship due to thermal expansion and volumetric change of the basaltic chamber. The term  $J_b$  is defined by:

$$J_b = 0.1 \left( \frac{\alpha_b g \kappa_b^2}{\nu_b} \right)^{1/3} \quad (2)$$

We can equate Eq (1) with the flux which is passes through the bottom of he silicic magma layer, thus:

$$F_b = \rho_s c_s J_s (T_i - T_s)^{4/3} \quad (3)$$

Now equating Eqs (1) and (3) and solving for  $T_i$ , we can find the interfacial temperature:

$$T_i = \frac{(T_b + y T_s)}{(1 + y)} \quad (4)$$

where:

$$y = \left( \frac{\rho_s c_s J_s}{\rho_b c_b J_b} \right)^{3/4} \quad (5)$$

Conservation of enthalpy through the basaltic and silicic melt layers require that:

$$\frac{dT_b}{dt} = - \frac{\left( \frac{J_b}{D} \right) (T_b - T_i)^{4/3}}{\left[ 1 - L_b c_b^{-1} \chi'_b (T) \right]} \quad (6)$$

$$\frac{dT_s}{dt} = \left( \frac{J_s}{a} \right) \left\{ \left[ (T_i - T_s)^{4/3} - (T_s - T_m)^{4/3} \right] - \frac{da}{dt} \left[ T_s - T_m L_s c_s^{-1} \chi'_b (T_s) \right] \right\} \times \left[ 1 - L_s c_s^{-1} \chi'_s (T_s) \right] \quad (7)$$

where  $\chi'(T_s)$  is the derivative of the crystal content within the silicic magma with respect to temperature. If we examine the terms on the right hand side for Eq.(7), we note that the first term simply scales the argument to one dimension. For the terms within the curly brackets we see that the first term is the net flux through the basalt/felsic thermal interface, and the second term is the net flux through the felsic layer, with a term to account for the amount of crystallized material within the melt. The multiplicative term is to account for the heat of crystallization, which modifies the fraction of the non-crystalline volume, which in turn is being affected by the temperature change. Note that Eq. (7) is representative of the change in temperature of the felsic magma with respect to time through three temperature phases; raising the temperature to its melting point, through the melting phase, then above its melting temperature.

The conservation of heat at the solid/melt interface above the felsic magma is given by:

$$\frac{da}{dt} = \frac{F_s}{H_s} \quad (8)$$

where:

$$F_s = \rho_s c_s J_s (T_s - T_m)^{4/3} \quad (9)$$

and:

$$H_s = \rho_s [c_s (T_m - T_0) + L_s] \quad (10)$$

Finally, the basaltic magma gains enough crystallinity through heat loss that the magma viscosity increases, convection slows, and eventually stops. At that point, the heat transferred from the basaltic magma is strictly conductive which is at a far lower rate

than the heat lost from the silicic magma layer as a consequence of convection. The silicic layer cools and crystallizes, and ultimately convection ceases.

An additional equation which is not mentioned in this sketch is the equation for the Rayleigh number for the felsic melt, which must exceed a critical value on the order of  $10^3$  for convection to be initiated. That equation is:

$$Ra_s = \frac{\alpha_s g (T_i - T_m) a^3}{\kappa_s \nu_s} \quad (11)$$

Since their theory is initiated with a convecting basaltic magma chamber, the Rayleigh number of basalt is not stressed in either of the H&S papers, although is given in Fig 4c of H&S1988a as initially  $10^{14}$ .

#### *Application to the Geological Process*

Huppert and Sparks then applied their theoretical description to the case of basaltic and felsic liquids. They addressed a standard case first, then varied the main parameters to arrive at four other cases. This report will, for the most part, address only the standard case.

The standard case consists of a basaltic sill with a thickness of 500 m and an initial ambient temperature of  $1200^\circ\text{C}$ , emplaced into continental crust with a fusion temperature of  $850^\circ\text{C}$  and an initial temperature of  $500^\circ\text{C}$  (i.e. lower crust, cf., Glazner and Ussler, 1988). The granitic roof rock will melt, and initially transfer heat via conduction. Once the critical Rayleigh number is reached, (which they calculate to be 2000), convection commences in the silicic layer. This occurs after a period of 7 days

where the silicic magma layer is a thickness of 1.2 m. After one year's time, the Rayleigh number is on the order of  $10^6$ , and the sill thickness is approximately 7 m. The system is active for approximately 90 yrs at which time the basalt, which has systematically gained crystallinity and lost heat, ceases to convect. Concurrently, the silicic magma chamber has a temperature of  $934^{\circ}\text{C}$ , and a Rayleigh number of about  $10^9$ . After approximately 1000 yrs the Rayleigh number of the felsic magma will fall below  $10^3$  (due to gained crystal content and loss of heat), and convection will terminate. The thickness of the felsic magma is around 300 m at the time of cessation.

### **Discrepancies of Theoretical Model with Nature**

During the course of this investigation, I have encountered a number of elements in the Huppert and Sparks theoretical model which seem incongruous to the natural geologic processes. These disparities will be addressed individually, and are the focus of the remainder of this paper.

#### *Convection*

Huppert and Sparks set their model based on the vigorous convection which occurs in both the aqueous solution and the PEG wax from their experiments. Nevertheless within the H&S1988a paper they state:

“It is possible, however, that  $R_s$ , which is small both initially and after a long time, never exceeds the critical value”  
This view is echoed by Brandeis and Marsh (1989), Bergantz and Dawes (1994), Brown, et.al (1995), and Barboza and Bergantz (1996). All four of these papers express the unlikelihood of vigorous convection in nature. In fact, it is the conclusion of Brandeis and Marsh (1989) that if convection occur in these circumstances at all in nature, that it is

short-lived, and the result of superheating. This conclusion was based on paraffin experiments performed by the aforementioned authors, and the lack of geologic evidence.

Furthermore, in H&S1988b a series of equations are derived (Eqs 27a-32b) in which they change notation so as to utilize dimensionless variables. The crucial point of this was to prove that for equal temperature differences convective fluxes are much larger than conductive fluxes (as demonstrated in Fig 8 of H&S1988b). This holds true as long as the phase diagrams are similar for the substances being compared. However, the phase diagram for the aqueous solution in the experiment is a vastly different from that of basalt (McBirney, 1993). In a simple qualitative argument, one can see that the simple eutectic of water will insufficiently simulate basalt.

### *Crystallinity*

To account for crystallinity within the experiments, Huppert and Sparks refer to experiments performed by Campbell and Turner (1987), in which they utilize two aqueous solutions and their solid equivalent. Yet in the Campbell and Turner paper, the aqueous solutions assimilated the melt in almost all of the experiments they performed. Thus it seems inappropriate to compare the Campbell and Turner experiments to the Huppert and Sparks experiments since the results were quite different. Additionally, Bergantz and Dawes (1994) point out (in reference to H&S1988a):

“The PEG wax used as country rock has a melting interval between 37 and 40°C and at the start of the experiment was isothermal with a temperature of 20°C. Thus, in no circumstances could the underplated “basalt” solution crystallize in the models, and hence no rheological penalty for crystallization is possible”

### *Prantl Numbers*

Naturally, some of the most convincing evidence is quantitative. The Prantl number, which by definition is:

$$\text{Pr} \equiv \frac{\nu}{\kappa} \quad (12)$$

is essentially a ratio of a fluids' momentum to it's heat. Thus, a fluid which diffuses heat faster than it does momentum will have a small Prantl number, and a fluid with a large momentum compared to the way it diffuses heat, will have a large Prantl number. Since the Prantl number is dimensionless, substances compared should be at least within the same magnitude of one another. This, however, was not the case when comparing nature to experiment, as seen in Table 1:

	<b>Kinematic Viscosity <math>\nu</math> (in <math>\text{cm}^2</math> <math>\text{s}^{-1}</math>)</b>	<b>Thermal Diffusivity <math>\kappa</math> (in <math>\text{cm}^2 \text{s}^{-1}</math>)</b>	<b>Prantl Number (Pr) (dimensionless)</b>
Basalt	$1.00 \times 10^3$	$8.00 \times 10^{-3}$	$1.25 \times 10^5$
Water <sup>12</sup>	$4.13 \times 10^{-3}$	$1.61 \times 10^{-3}$	2.55
Felsic	$1.00 \times 10^6$	$8.00 \times 10^{-3}$	$1.25 \times 10^8$
PEG 1000 wax <sup>34</sup>	$1.74 \times 10^7$	$5.24 \times 10^4$	$3.32 \times 10^2$

**Table 1**

The comparisons of basalt/ water, and felsic/ PEG 1000 wax are different by five and six orders of magnitude, respectively. This shows that for these substances, the mechanical and thermodynamic properties completely different, hence the analogy of experiment to nature is flawed.

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<sup>1</sup>Note: the physical constants of water were employed because it is believed by the author that they are only negligibly different from the sodium nitrate/ aqueous solution.

<sup>2</sup>Physical constants for water are from the CRC Handbook of Chemistry and Physics, 76th Ed.

<sup>3</sup>Kinematic Viscosity is from the CRC Handbook of Chemistry and Physics, 65th Ed.

<sup>4</sup>Thermal diffusivity was calculated from  $\kappa=k/\rho c$ , where the density of PEG 1000 is from the CRC Handbook of Chemistry and Physics, the thermal conductivity, k, is from DiGuillio and Teja (1990), and the specific heat was found on the Web at: <http://chemfinder.camsoft.com>

### *Lack of Evidence in Nature*

Finally, an important test of a model is to look into nature to determine if there is sufficient evidence which supports the theory. For a basaltic magma chamber positioned below a silicic magma chamber, one expects to commonly see gabbros beneath a migmatitic source regime. Yet the occurrence of this is rare, according to

Brown et al.(1995) who state:

“Much of the popularity of the emplacement-of-basalt-to-melt-the-crust model relies on the experiments of *Huppert and Sparks* (1988), but it is uncertain exactly how these experiments can be related to certain processes and geologic timescales ( *Bergantz and Dawes*, 1994), in particular, given the reliance of the model on turbulent convection within the basalt and bulk melting of the roof zone, for which widespread geologic evidence is scarce.”

### **Concluding Remarks**

The flaws in comparing these experiments to nature seem quite clear. Yet H&S1988a (and H&S1988b, as well), continues to be a very well cited article. In fact, on average, H&S1988a is cited eighteen times per year. The obvious question becomes then, why is it cited so often if it is inapplicable to the geologic process? To help answer this question I randomly chose articles written in 1996 and 1997, all of which had cited H&S1988a in their papers. This was the result: out of twelve articles randomly picked, nine of them, Warren and Ellis (1996), Guffanti et al. (1996), Kincaid and Silver (1996), McCarthy and Douce (1997), Weinberg (1996), Henk et al. (1996), Galan et al. (1996), Keay et al. (1997), McMurcheon et al. (1996), cite H&S1988a utilizing it only as a conductive heat transfer argument. Two additional articles, Deleris et al. (1996), and Bagdassarov et al. (1996), blatantly misquoted H&S1988a by stating that H&S1988a assumes assimilation of the melt into one or both of the magmas. One article, Cardoso and Woods (1996), quoted H&S1988a as a convective model, and utilized its' equations

to form a revised theory of convective heat transfer through a melting roof. However since these two individuals are from the same department at Cambridge as H.E. Huppert, I find it moderately difficult to believe that they were not somewhat influenced.

It is important to reiterate that this model has been consistently over-cited. Although the theoretical model from H&S1988a is valid for certain types of convecting fluid, and perhaps can be utilized for modelling aqueous solutions, the model is completely invalid for geologic processes. In light of this information, a reexamination of how and when this model is applied is a worthwhile consideration.

## Appendix A Nomenclature

<b>Symbol</b>	<b>Definition</b>	<b>Symbol</b>	<b>Definition</b>
$F_b$	convective flux from the basaltic magma	$\kappa$	thermal diffusivity
$a$	thickness of the silica layer	$\nu$	kinematic viscosity
$L_s$	latent heat of fusion of silica	$T_i$	interface temperature
$L_b$	latent heat of crystallization of basalt	$g$	acceleration due to gravity
$\alpha$	coefficient of thermal expansion	$T_m$	temperature at which the roof rock melts ( $T_m=T_i$ , at initial conditions )
$\rho$	density of the magma	$T_b$	temperature of the basalt layer
$c$	specific heat of the magma	$T_s$	temperature of the well mixed silica
$J$	defined by Eq (2)	$D$	thickness of the basalt layer
$H_s$	heat needed to raise unit volume of solid crust from its far-field temperature to its melting temperature and also to melt it	$\chi$	fraction of crystal content within the magma
		$y$	defined by Eq (5)
<b>Subscript</b>	<b>Definition</b>	<b>Subscript</b>	<b>Definition</b>
s	silica	o	initial condition
b	basalt	i	interface
sol	solid		

## References

- Bergantz, G.W. & Dawes, R., 1994. Chapter 13 of Magmatic Systems, Edited by M.P. Ryan
- Brown, N., Rushmer, T., Sawyer, E.W., 1995. Introduction to special section: Mechanisms and consequences of melt segregation from crustal protoliths. *J. geophys. Res.*, **100**, 15,551-15,563
- Brandeis, G., & Marsh, B.D., 1989. The convective liquidus in a solidifying magma chamber: a fluid dynamic investigation. *Nature* **339**, 613-616
- Huppert, H.E., & Sparks, S.J., 1988a. The generation of granitic magmas by intrusion of basalt into continental crust. *J. Petrology*, **29**, 599-624
- Huppert, H.E., & Sparks, S.J., 1988b. Melting the roof of a chamber containing a hot, turbulently convecting fluid. *J. Fluid Mech.* **188**, 107-131
- DiGuilio, R. & Teja, A.S., 1990. Thermal conductivity of poly(ethylene glycols) and their binary mixtures. *J Chem. Eng. Data* **35**, 117-121
- Warren, R.G., Ellis, D.J., 1996. Mantle underplating, granite tectonics, a metamorphic P-T-t paths. *Geology* **24**, no. 7, 663-666
- Guffanti, M., Clynnne, M.A., Muffler, P.J.L., 1996. Thermal and mass implications of magmatic evolution in the Lassen volcanic region, California, and minimum constraints on basalt influx to the lower crust. *J. geophys. Res.*, **101**, 3003-3013
- Kincaid, C., Silver, P., 1996. The role of viscous dissipation in the orogenic process. *Earth Planet. Sci. Lett.* **142** 271-288
- McCarthy, T.C., Douce, A.E.P., 1997. Experimental evidence for high-temperature felsic melt formed during basaltic intrusion of the deep crust. *Geology* **25**, no. 5, 463-466
- Henk, A., Franz, L., Teufel, S., Oncken, O., 1997. Magmatic underplating, extension, and Crustal Reequilibrium: Insights from a cross-section through the Ivrea zone and Strona-Ceneri zone, Northern Italy. *J. Geol.* **105**, 367-377
- Galan, G., Corretge, G.L., Laurent, O., 1997. Low-potassium vaugnerites from Gueret (Massif Central, France). Mafic magma evolution influenced by contemporaneous granitoids. *Min. Pet.* **59**, 165-187
- Keay, S., Collins, W.J., McCulloch, M.T., 1997. A three-component Sr-Nd isotopic mixing model for granitoid genesis, Lachlan fold belt, eastern Australia. *Geol.* **25**, no.4 307-310
- McCutcheon, S.R., Anderson, H.E., Robinson, P.T., 1996. Stratigraphy and eruptive history of the late devonian Mount Pleasant caldera complex, Canadian Appalachians. *J. Geol.* **104**, 34-44
- Weinberg, R. F., 1997. The disruption of a diorite magma pool by intruding granite: The Sobu Body lake, Ladakh batholith, Indian Himalayas. *J. Geol.* **105**, 87- 98
- Barboza, S.A., Bergantz, G.W., Dynamic model of dehydration melting motivated by a natural analogue: applications to the Ivrea-Verbanò zone, northern Italy. *Special paper- Geol. Soc. Am.* **315** 23-31
- Bagdassarov, N.S., Dorfman, A.M., Dingwell, D.B., 1996. Modelling of melt segregation processes by high -temperature centrifuging of partially molten granites- I. Melt extraction by compaction and deformation. *Geophys. J. Int.* **127**, 616-626

- Huppert, H.E., 1986. The intrusion of fluid mechanics into geology. *J. fluid Mech.* **173**, 557-594
- Cardoso, S.S.S., Woods, A.W., 1996. Interfacial turbulent mixing in stratified magma reservoirs. *J. Volcanol. Geotherm. Res.* **73**, 157-175
- Deleris, J., Nedelec, A., Ferre, E., Gleizes, G., Menot, R.P., Obasi, C.K., Bouchez, J.L., 1996. The Pan-African Toro Complex (northern Nigeria): magmatic interactions and structures in a bimodal intrusion. *Geol. Mag.* **133**, 535-552